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Multivariate Power Law Modelling of Apparent Viscosity of Synthetic-Based Mud Used in High-Temperature Gas Wells Using Marsh Funnel and Mud Density Test Results

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Abstract

Despite intensive efforts to diversify the global reliance on fossil fuel, it remains one of the foremost sources of energy. This has led to production activities advancing into very difficult areas such as deep sea, and high-temperature/high-pressure wells. This in turn puts the drilling team in the spotlight as their failure would result in great loss. Rheology is the aspect of science that seeks to understand the behaviour of substances when subjected to different forces. Viscosity measures the extent to which a given fluid resists the tendency to flow. Lucky proposed a modification to the Bingham plastic model which when applied to two samples of synthetic-based muds performed better than the traditional Bingham plastic and power law models. The model included a dimensionless correction factor that increased the accuracy of the model. There is therefore need for more rigorous testing of the model and consequent adoption of the model for field operations within the range of applicability that the testing proves efficient for operations. Therefore, the impact of mud weight in the prediction of plastic viscosity, yield point or apparent viscosity is minimal. The model in this work was obtained for a specific synthetic based mud used in a high-temperature gas well and should be used within the specific conditions that apply to its derivation. The degree of applicability of the proposed model should be determined to ascertain the conditions and range of values within which it gives the best possible predictions to optimize its use for field operations.

Keywords: Power law; Gas well; Marsh funnel; Mud density test; Viscosity

Introduction

Global population explosion has led to increase in the demand for energy globally. Despite intensive efforts to diversify the global reliance on fossil fuel, it remains one of the foremost sources of energy. This has led to production activities advancing into very difficult areas such as deep sea, and high-temperature/high-pressure wells. This in turn puts the drilling team in the spotlight as their failure would result in great loss. The difficulties faced during a drilling process is managed by modifying the drilling fluid.

The wide time interval between successive tests performed on the drilling mud to give comprehensive information creates an operational gap. This adds another layer to the challenge of making decisions during drilling. Because of this, a lot of work has been done to develop standard ways of predicting the properties of drilling mud in a repetitive and reliable manner. Apparent viscosity, among other properties, help to understand the behaviour of fluids [1].

Literature Review

Rheology is the aspect of science that seeks to understand the behaviour of substances when subjected to different forces. Viscosity measures the extent to which a given fluid resists the tendency to flow [2]. Plastic viscosity, apparent viscosity, gel strength and yield point are the properties commonly measured. Though all rheological properties contribute to the performance of a fluid, some take less time to measure while others take more time. Mud density and marsh funnel viscosity are the easiest and fastest to measure. They can be measured as often as every 15-20 minutes. This ease in the measurement of these two properties is responsible for the quest to extract more information from them [3].

Bingham did a foundational work in explaining the behaviour of some fluids. The reason behind the popularity of the Bingham model is that it is simple yet useful for gaining insight into fluid behaviour. The challenge with the model is that at low shear rates, it gives inaccurate results. Another model that is also popular is the Power Law model. It tried to address the limitation of the Bingham model but fails to give accurate results for fluids that are better described by Bingham model [4,5].

Even though there is a lot of work on this topic, simple, yet useful models are still scarce. Figure 1 shows the dimensions of a marsh funnel.

In a review of models for drilling fluid rheology, many models were complex and were difficult to apply in the field, while others that were simple had limitations [6].

Ahmadi studied the interaction of three variables on density at down-hole conditions. The PSO-ANN model gave the most favourable result. Rahmati and Tatar used RBF-NN to describe the behaviour of four different types of mud. In the works by Alsabaa and

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Agwu, the ANN gave the best description of the behaviour of the synthetic-based and oil based mud studied, respectively [7].

Alsabaa presented an ANN based model for real time monitoring of an all-oil mud system and derived empirical correlations for estimating rheological parameters of all-oil mud with ANN. Gomaa developed ANN models to predict rheological properties of high overbalanced water-based drilling fluid. Gowida deployed ANN models to determine rheological properties of high Bentonite mud (water-based mud).

Alizadeh studied three models for predicting drilling fluid density at HPHT conditions with PSO-LSSVM being the best of the models [8].

The model compared favourably with six other models derived from other studies. Gowida developed models to predict yield point, plastic viscosity, and apparent viscosity of Calcium Chloride ($CaCl_2$) brine-based drill-in the fluid. The input parameters used were mud density and marsh funnel viscosity [9].

Aljubran derived a laboratory based machine learning model to estimate drilling fluid properties using drilling mud formulations.

Alsabaa employed Adaptive Neuro-Fuzzy System (ANFIS) to develop models for obtaining determining rheological properties of the invert emulsion. Elkatatny used ANN to predict the rheological properties of NaCl mud [10].

Ofoche applied machine learning algorithms to the marsh funnel geometry to model and predict rheological profiles across the marsh funnel. Zheng developed a model based on Herschel-Bulkley model to predict the dynamic force and the marsh funnel time of the tested fluid. Pitt obtained a model, based on numerical simulation, for determining effective viscosity from Marsh funnel time and mud density. Almahdawi obtained an empirical equation that was derived from the analyses of experiments.

In Liu four real-time measurement technologies were studied and pipe viscometer was proved to give reliable measurement but was not convenient for field use. Leusheva studied models describing the behaviour of barite free water-based mud and concluded that Herschel-Bulkley model was the best for the study. The studies by Onuh and Hamed also found the Herschel-Bulkley to give the best result for their studies respectively.

Akintola proposed a model for the prediction of plastic viscosity of water based drilling fluid. The model required three variables for predicting the plastic viscosity. Ofoche found a progressive increase in the accuracy of their rheological models from low to high shear rates like the progression of accuracy observed in conventional rotational rheometers. Zhang found that the white oil-based mud was described by the Bingham plastic model. This indicates that the Bingham Plastic Model can be used to describe the behavior of some oil-based muds.

Lucky proposed a modification to the Bingham plastic model which when applied to two samples of synthetic-based muds performed better than the traditional Bingham plastic and power law models. The model included a dimensionless correction factor that increased the accuracy of the model. The Herschel-Bulkley model was used as the baseline for the performance of the models. Elkatatny applied a self-adaptive differential evolution technique to optimize the ANN variables of a NaCl-water-based drill in fluid. The apparent viscosity model from this technique outperformed the previous models highlighted in the study.

Yiwan studied the rheology of drilling fluid using different measurement tools and noted that rotary viscometer was more reliable in characterising drilling fluid qualitative rheology than marsh funnel. It was further noted that drilling fluids with similar rheology at high shear rates could have dissimilar rheology at low shear rates.

The marsh funnel is the oldest field instrument used for checking the consistency and viscosity of drilling fluid. It finds application in both the oil industries as well as construction industries. Limited investigations have been made to propose an easy, fast and reliable fluid viscosity analysis in terms of marsh funnel discharge flow time.

The study developed an analytical model based on Torricelli's theorem and was verified numerically and experimentally. It was found to give the required engineering accuracy but was limited to Newtonian fluids. In an experimental study, Sidik evaluated the models using eight (8) samples of water-based mud. It was observed that the model proposed by Almahdawi outperformed the Pitt model. The study by Ofoche agreed with that of Pitt on the time for a fluid of negligible viscosity to flow through the marsh funnel (24.5 seconds). The study proposed a polynomial equation for predicting dial readings [11].

Liu confirmed from their research that there is presently no industry standard handbook for real-time drilling fluid measurement. Sadrizadeh also asserted that limited investigations have been made to propose an easy, fast and reliable fluid viscosity analysis in terms of marsh funnel discharge flow time.

To this end there is still need for concerted research efforts to work towards the availability of quality research in this area to foster the standardization of real time drilling fluid measurement and determination using mathematical models.

Viscometers are used to obtain dial readings at different revolutions per minute which are then converted to the various rheological parameters required by means of appropriate equations as given in equations (1-6). These parameters are used in the various mathematical models to understand the behavior of the fluid under study.

$$PV=R_{600}-R_{300} \quad (1)$$

$$YP=R_{300}-PV \quad (2)$$

$$R_{300}=YP+PV \quad (3)$$

$$PV=R_{600}-(YP+PV) \quad (4)$$

$$R_{600}=2PV+YP \quad (5)$$

$$AV=1/2\ (2PV+YP) \quad (6)$$

Power-law model: It describes the relationship between the shear stress and shear rate of a fluid using three variables. $\Gamma=K^n$ (7)

Where: Γ=Shear stress

γ=The shear rate

K=Consistency coefficient

n=Flow behavior index

Herschel-Bulkley model: This model involves the addition of the yield stress variable to the power law model to accommodate the effect of yield stress. $\Gamma= \Gamma_y+ K\gamma^n$ (8)

Where: Γ_y=Yield stress

Bingham plastic model: This model indicates that a level of stress must be overcome before a fluid begins to flow. $\Gamma = \Gamma y + \eta\gamma$ (9)

Where:

η=Plastic viscosity in cp.

Casson model: The Casson model is more complex and involves the fluid's yield stress and the curvature of its flow curve. $\Gamma^{0.5}-\Gamma 0^{0.5} = K(\gamma^{0.5}-\gamma^{0.5})$ (10)

Where:

γ=Yield shear rate

Apparent viscosity equations for Bingham, power law and Herschel-Bulkley modes are given below.

Power law: $\mu_{app}=K\gamma^{n-1}$ (11)

Herschel-Bulkley: $\mu_{app}=(\Gamma_y/\gamma)+K\gamma^{n-1}$ (12)

Bingham: $\mu_{app}=\eta+((\Gamma_y/\gamma))$ (13)

Where:

η =Plastic viscosity in cp.

Pitt obtained a model (equation 14) for the determination of effective viscosity from Marsh funnel time and mud density. Almahdawi obtained an empirical equation (equation 15) that performed better than Pitt.

$\mu_{effective}=\rho(t-25)$ (14)

Where $\mu_{effective}$ is effective viscosity in cp; ρ is density in g/cm^3, and t is marsh funnel time in seconds.

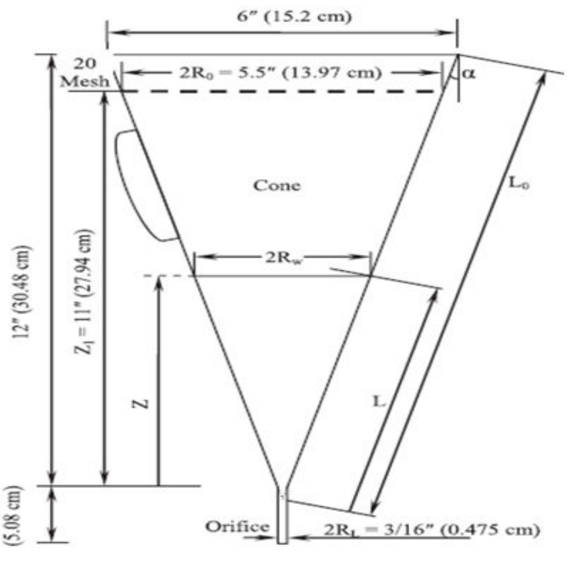

Figure 1: Marsh funnel.

Other works on apparent viscosity prediction include Elkatatnyet, Al-Khdheeawi, and Ofoche. Gowida and Al-Khdheeawi are machine learning models.

Methodology

Final Well Reports of four high-temperature gas wells from NOPIMS website were used. Data (marsh funnel viscosity, mud density, plastic viscosity, and yield point) of synthetic-based mud for three (3) wells (well 001, well 002, and well 003) were compiled to give a total of 65 data points. The procedure for the field measurements of the data used can be found in API. Multivariate Power Law regression was performed on the data using Python programming language to obtain a model that could determine the apparent viscosity of the mud. Statistical analysis including mean absolute error, mean absolute percentage error, mean square error, root mean square error, and r-squared were used to assess the performance of the model. Synthetic based mud data from the fourth well (well 004) was used to validate the model [12].

The predicted values of the apparent viscosity from the model were compared with those calculated using the viscometer readings as given in equation (6). Statistical analysis including mean absolute error, mean absolute percentage error, mean square error, root mean square error plastic model, power law model, and Herschel-Bulkley model were used to calculate the apparent viscosity of the mud from the validation fourth well. Models from the literature were also used to calculate the apparent viscosity. The results of the models from the literature and those used in the industry were compared to the multivariate power law regression model.

The multivariate power law regression gave an apparent viscosity formula given in equation (19): $\mu_{app}=0.216865(\rho * t)^{1.172192}$ (19)

Where; μ_{app}=Apparent viscosity in cp

ρ=Density in g/cm^3

T=Marsh funnel viscosity in seconds per quart

The power law and Herschel-Bulkley models were modified to give more accurate values of apparent viscosity by introducing a correction (multiplication) factor of 0.5 to the models to give:

$\mu_{app}= 0.5(K\gamma^{n-1})$ (20)

And $\mu_{app}=0.5^{*})+K\gamma^{n-1}$ (21)

The performance of the model in this study was analysed using the Root Mean Square Error (RMSE), Mean Absolute Percentage Error (MAPE or Average Absolute Percentage Error-AAPE), Mean Square Error (MSE), and Mean Absolute Error (MAE).

Generally, when comparing two mean absolute errors, a smaller value indicates a better model performance and a higher value indicates a poorer model performance. When comparing two MSE values, a larger value indicates a higher error margin while a smaller value indicates a smaller error margin. However, this error matrix can be affected by large error values within the data. When comparing two models using the RMSE matrix, a model with a lower RMSE value would generally perform better than a model with a higher RMSE value.

The relationship between mud weight and plastic viscosity, yield point and apparent viscosity was studied using scatter plot. Also, a scatter plot was used to observe the relationship between marsh funnel viscosity and plastic viscosity, yield point and apparent viscosity.

Discussion

Table 1 gives a summary of the statistics of the data used for the Power law regression and validation of the model. It can be seen from the tables that the range of values used for the regression are greater than those used for the validation. Table 2 gives the error analysis of the performance of the power law regression model on the validation data set. It can be seen from Figure 2 that the model from this work overlaps the actual apparent viscosity, but deviates at two points, when compared to industry models [13].

Regression		MW, sg	MF, sec/qt	PV, cP	YP, lb/100 ft^2	AV, cP
Validation	Min.	1.32	55	30	15	38.5
	Max.	1.56	129	56	41	75
	Range	0.24	74	26	26	36.5
	Min.	1.4	54	26	19	36
	Max.	1.58	65	36	31	49
	Range	0.18	11	10	12	13

Table 1: Statistical analysis of synthetic mud data used for study.

Model	MAE	MAPE	MSE	RMSE	R^2	R
Regression data	3.2424	6.52	15.0347	3.8775	0.7893	0.8884
Validation data	1.8932	4.5	6.3807	2.526	0.7246	0.8512

Table 2: Error analysis of the performance of the model obtained from this work.

However it showed a better overlap with the actual apparent viscosity much more than any other model in literature. Figure 3 shows that the correction factor resulted in the power law and Herschel-Bulkley models giving the exact measurements as obtained from the viscometer readings. Figures 4 and 5 show that the model from this work performs better than models in literature.

The mean absolute percentage error of the Power law and Herschel-Bulkley models were approximately 100% respectively. This indicated an overestimation about twice the actual values of the apparent viscosities of the mud. This highlighted the need to introduce a correction factor into the power law and Herschel-Bulkley models to correct the overestimation. After correction, the MAE, MAPE gave values of approximately Zero (0) for both the Power law and Herschel-Bulkley models indicating that the synthetic-based mud used for the high-temperature gas well under consideration was a Power law fluid. It can therefore be concluded that the correction factor introduced to the power law and Herschel-Bulkley models be used when considering power law fluids only.

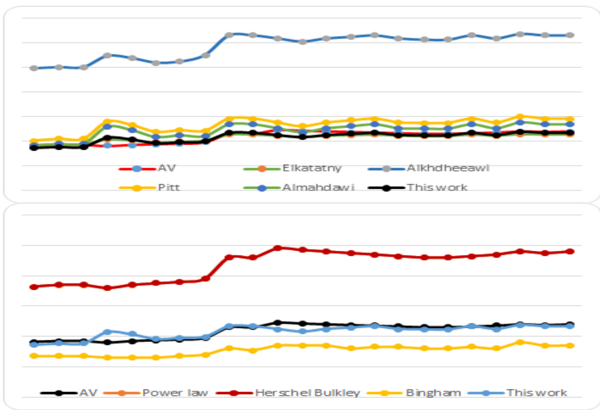

Figure 2: Comparison of this work with industry models and models in literature.

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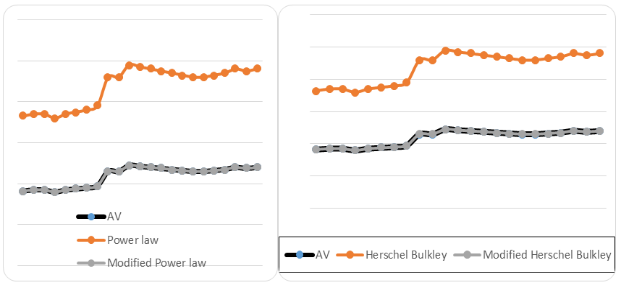

Figure 3: Comparison of power law, Herschel-Bulkley models the modified models.

It is worth noting that unlike other models such as the experimental and numerical model referred to in this work; none of them categorically outperformed machine learning models [14].

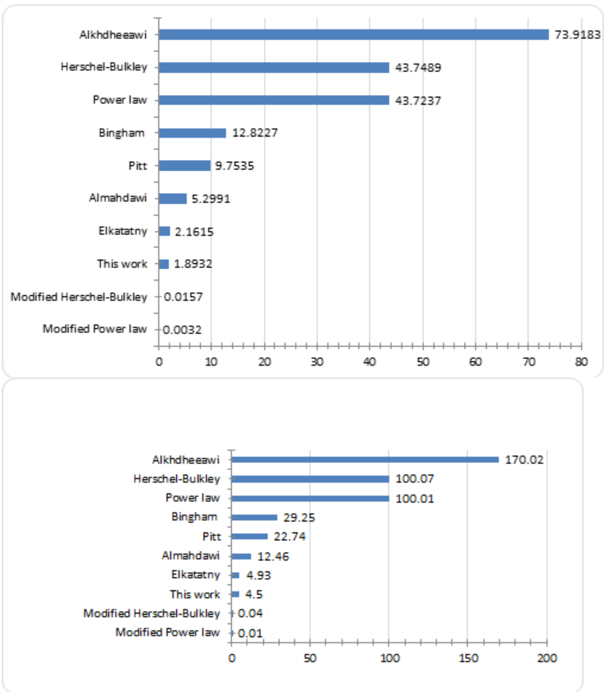

Figure 4: Comparison of mean absolute percentage error of different models.

However, the model obtained from this work outperformed both experimental, numerical as well as machine learning model. It is worthy of note that the model by Elkatatny also outperformed other models but did not outperform the model from this work. This model not only meets the simplicity required for it to find usefulness in the field during oil and gas operations but also meets the accuracy level required for it to find technical relevance in the field.

There is therefore need for more rigorous testing of the model and consequent adoption of the model for field operations within the range of applicability that the testing proves efficient for operations. Figure 6 shows there is very little relationship between the mud weight and plastic viscosity, yield point or apparent viscosity. Therefore, the impact of mud weight in the prediction of plastic viscosity, yield point or apparent viscosity is minimal.

Figure 6 shows there is very little relationship between the mud weight and plastic viscosity, yield point or apparent viscosity. Therefore, the impact of mud weight in the prediction of plastic viscosity, yield point or apparent viscosity is minimal. The plot of marsh funnel viscosity versus plastic viscosity, yield point and apparent viscosity (Figure 7) gives a better relationship compared to mud weight. The relationship between the marsh funnel viscosity and the three variables is similar. This can be explained by the fact that the three variables are related by a simple mathematical relation.

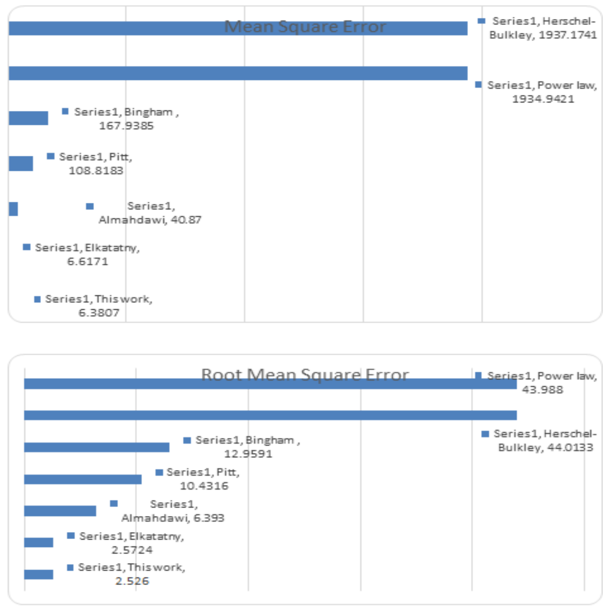

Figure 5: Comparison of mean square error and root mean square error of selected models.

The plot of marsh funnel viscosity versus plastic viscosity, yield point and apparent viscosity (Figure 7) gives a better relationship compared to mud weight.

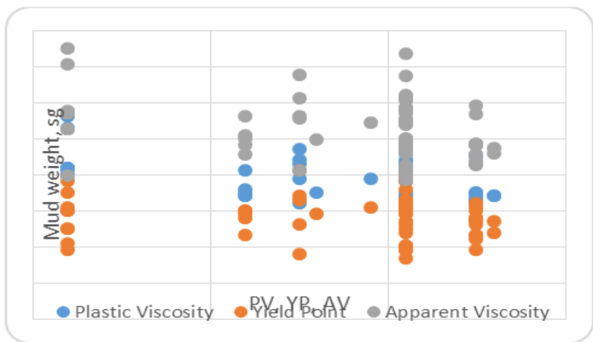

Figure 6: Plot of mud weight versus plastic viscosity, yield point and apparent viscosity.

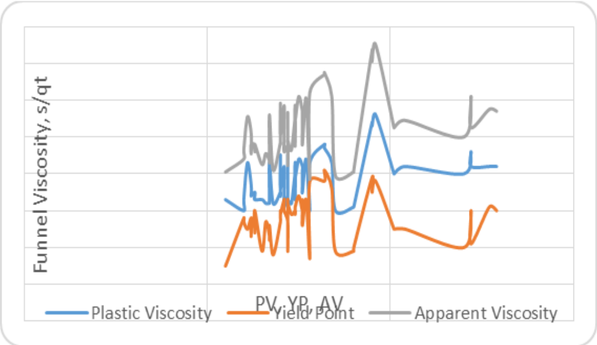

Figure 7: Plot of marsh funnel viscosity versus plastic, apparent viscosities and yield point.

The relationship between the marsh funnel viscosity and the three variables is similar. This can be explained by the fact that the three variables are related by a simple mathematical relation.

Application of models

Correlations and equations that provide an alternative means of determining scientific measurements have always found usefulness in scientific and engineering applications. Especially in the oil and gas industry where there is a high level of uncertainties that characterize down-hole measurements, it is needful to have correlations that are as close to the actual measurements provided by machines to provide backups in the event of unforeseen circumstances.

Such correlations if properly analyzed can provide a safe fall back in the events of equipment failure, as well as provide a way of detecting anomalies in the measurement to detect possible challenges or human errors in measurement. This model provides a framework for real time monitoring of the rheological properties of the drilling mud under field conditions. It can be fine-tuned with a larger volume of data to give a higher level of accuracy. This would extend the usefulness of the marsh funnel.

Conclusion

The model in this work was obtained for a specific synthetic based mud used in a high-temperature gas well and should be used within the specific conditions that apply to its derivation. The degree of applicability of the proposed model should be determined to ascertain the conditions and range of values within which it gives the best possible predictions to optimize its use for field operations. More field data should be used for the regression and validation to cover a wider range of values and ensure the universality of the derived model. It is suggested that a different experiment as simple as the marsh funnel test that results in another variable that can be combined with the marsh funnel viscosity to describe the behavior of drilling fluids.

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