



Geometry and Proportion of Conical Domes' Plans in Iran: Reviewing Case Studies

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Abstract

The aim of this paper is to identify descriptive and quantitative features of conical domes to introduce geometry and proportion of their plans through analytic reviewing of case studies. Since geometry and proportion have been studied in conical domes, plans of conical domes need more attention in terms of geometry and proportion. Authors hypothesized that with introducing a new Ratio, analyzing geometry would be possible. This Ratio revealed nearly the value of $\sqrt{2}$ or $\sqrt{3}$ which is one of the most important Ratios in Islamic architecture. Authors have presented diagrams of superimposed squares and circles to suggest principles of designing with such Ratios. All kinds of plans whether circles or polygons can be designed based on these diagrams since circle is the basic geometry in Islamic architecture. Studying conical domes is important since studies show that their construction has lasted until Qajari era and may continue despite advances in other kinds of domes. Such studies strengthen the connection between past and present architecture of Iranian architecture.

Keywords: Conical domes; Plan form; Ratio; Proportions; Geometry

Introduction

Islamic art favors geometry in general and geometrical proportion in particular which carry great importance in the design process of Islamic art [1]. Islamic architecture has long been known as the embodiment of mathematical and geometrical premises [2]. In an architectural monument all dimensions, both in its integrity (height, length and width) and in its components (including geometrical surface patterns), are interrelated and never divorced from geometry [3]. Geometry as science for selection of structural dimensions such as height, length and width of the building and its structural elements governs the structural behavior of the building, the behavior that follows the geometry. The right geometry makes the building behave correctly [3]. In Persian architecture, it is geometry that provides diverse stylistic developments for constructions and designs; not only to serve a function, but also to evoke an emotional response by harmonization of the constructional elements, such as domes and columns and decorative elements [1]. Seyyid Hossein Nasr argued that geometry and rhythm manifest a doctrine of unity which is central to Islam, upon which Islamic art developed based on mathematical ratios and proportions which represent the very heart of Islam [4].

On the other hand, geometry involves proportions and prime roots that are considered the most beautiful proportions (the proportions of beauty) [4]. Proportion in architecture can be said to be "an harmonious relationship between the parts, with and within the whole" [3]. In architecture, the role of the proportioning system is to create a set of visual relationships between all the different parts of a building, and between the parts and their whole. The aim is to provide a sense of order in the overall structure [4]. Geometrical analysis of many Persian historic buildings has proven that a complete knowledge of proportions was widely used in Persian architecture [5] and it was the basis of Persian aesthetics [3].

Regarding studies related to geometry and proportion of domes, some papers [2-4,6-16] are worth considering. As can be seen, much attention has been paid to geometry and proportion of domes and their patterns but not that much to geometry and proportion of Islamic plans [6], especially those of conical domes which have been ignored compared to other kinds of domes. Moreover, plans of conical domes

are more likely to have special Ratios compared to pointed and bulbous domes mainly due to two reasons. First, technological and architectural advances has caused pointed and bulbous domes to be more various in geometry and proportion of plan forms. Second, pointed and bulbous domes have often been used in religious complexes than in single funerary buildings where the proportion and geometry of the whole complex becomes more important. In fact, such proportion may seem unique in the plans of conical domes. Authors hypothesized that having a Ratio in plans of conical domes does not seem inconceivable since geometry is apparent in other parts of these domes. We tend to examine the layout of these plans through samples to test our hypothesis. Accordingly, this paper is arranged into three parts: 1) studying descriptive information of case studies: morphology, plan form and their origin 2) Defining new Ratios based on quantitative features to test our hypothesis 3) introducing principles of drawing and designing.

Origin, History and Types of Conical Domes

Regarding the evolution of conical domes, double-shells are followed by one-shell domes and were mostly used due to climatic conditions like heavy raining [17]. Accordingly, these domes were first used in the north of Iran with such climatic conditions [18] to protect the internal shell from humidity [19]. Moreover, the interior shell could expose ornamental decorations with its proportionate dimension and the exterior shell could be designed higher to add significance to the building [19]. The triple-shell demonstrates the advancement of structural design by adding a third shell to enhance its stability [8]. The third shell provides more resistance against forces [20]. In the conical roof (shell) cases, stone slabs or brick layers rested on the lower roof

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(shell) with some internal voids for reducing the weight and protecting the lower parts [7].

Regarding different eras, there are numerous innovative architectural contributions made during Seljuks among which is advancing the use of conical domes [21]. Some of the most enduring signs of Seljuk architecture are the distinct types of polyhedral and conical domes which still stand in Iran as seen in Figure 1. These edifices are commonly well-known as a primitive architectural formula for the Islamic funerary buildings topped with conical and polyhedral shells which mainly appeared in the Seljuk period [8] (Figure 1).

During the Ilkhanid period, the construction of conical and polyhedral domes decreased considerably in comparison with the Seljuk era, but still there were two specific architectural achievements during this period, small brick connectors and extensive use of turquoise tile works as can be seen in Figure 2 [8].

The construction of conical and polyhedral domes became less important during the Timurid period due to two main reasons, by developing the use of pointed domes and with the introduction of the bulbous style [8]. It should be noted that these domes have also been built in hot-arid areas like Qom in this era. Its reason is mainly attributed to the fact that some great conical domes have been destroyed by sunnis¹ in Ilkhanids era which lead to the usage of conical domes as the symbol of shias² buildings. That is why the usage of these

domes became common in Qom and kashan after Ilkhani era without having any congruence with hot and dry climate of these cities [18]. Case studies of Timurids era show the emergence of polyhedral and pyramidal domes in religious complexes and hot-arid areas. Figure 3 shows case studies of Timurid era [22].

Still we can see the trace of these domes in different eras which surely shows their popularity among Muslims [23]. Another important point is that conical domes have appeared in religious complexes afterward and not as an individual funerary building. In some cases, the sanctuary's proportion can easily be addressed by conical domes and there is no need to construct domes with larger spans or heights. Shahzade Ebrahim in Kashan is among one of these complexes as can be seen below. This conical dome has been designed in a site with a lot of cypresses and near one of Kashan's old gates to attract attention when entering city. Therefore polyhedral dome is in complete agreement with required dimension, the shape of cypresses and location of the complex in city. That is why their construction continued in some cases even after Ilkhanids era (Figure 4).

Furthermore, the construction of conical domes have made progresses in following eras including the increased number of hedrons³ and increased height of drums as can be below Figure 5.

Case Studies' Information

Twenty-five samples of conical domes, which were built in Iran,

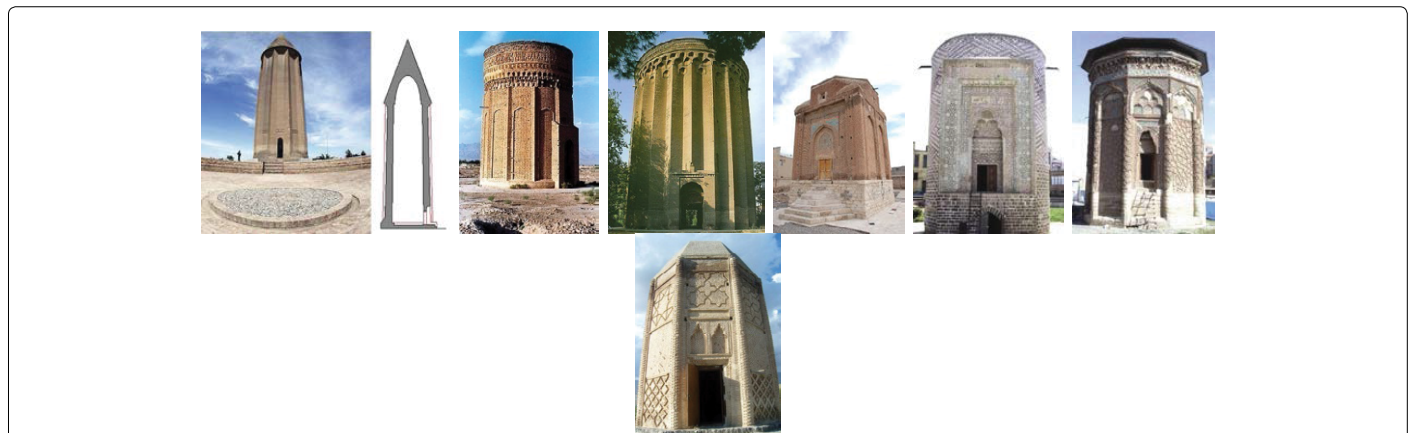


Figure 1: Case studies of Seljuks era. From left to right: Qabus dome [photo: 6]. Mehmandust [photo: <http://ghiasabadi.com/pep/tower-of-mehmandust.html>]. Toqrol [<http://www.irantouronline.com/tehran/historical-places/toqrol-tower-ray.html>]. Sorkh, Seh gonbad, Kabud [photo: <http://en.wikigogo.org/en/168827/>]. f) Damavand [photos:31].

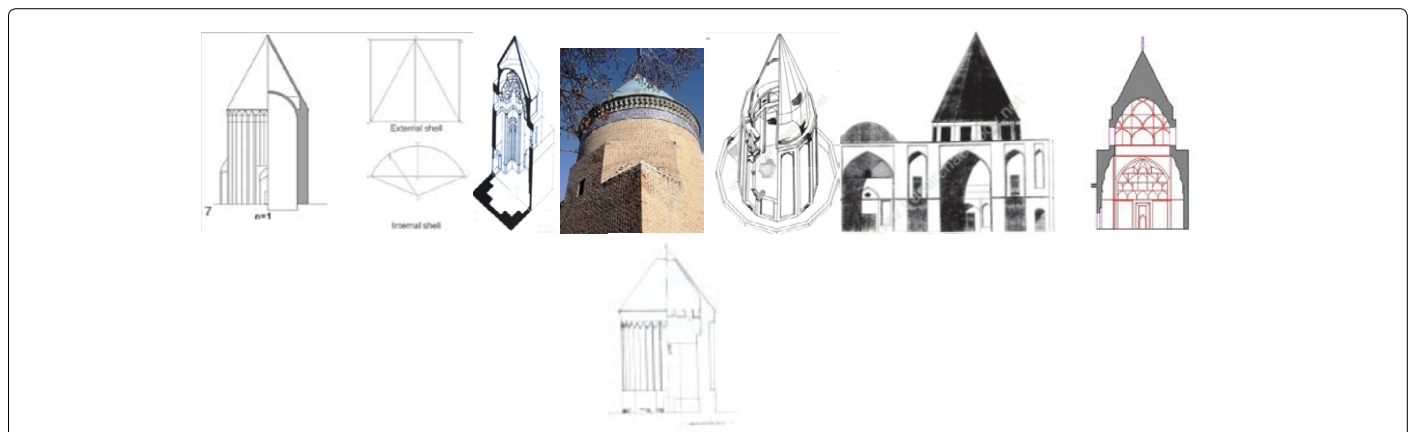


Figure 2: Case studies of Ilkhanids era. From left to right: Ala eddin dome[photo: 6]. Sheikh abdosamad [photo: 30]. Hamdollah mostofi [photo: http://en.wikipedia.org/wiki/Hamdallah_Mustawfi]. khaje asilodin [photo: 2]. Ali ebne jafar [photo: 2]. Baba qasem [photo: 6]. Radkan [photos:29].

were subjects of this analysis. Authors examine plans of conical domes to test their hypothesis. Tables 1 and 2 show the descriptive and quantitative features of case studies respectively. In Table 2, exterior diameter (E) shows either diameter of the circular plan or diameter of the circle that surrounds polygon or has been surrounded by polygon. Moreover, O Ratio (external to internal diameter Ratio) is introduced to reveal geometry and proportion of plans which is the purpose of this paper. M and N Ratios help show the overall form of conical domes. In Table 2, dimensions are based on plans acquired from different papers. These plans have been drawn and dimensioned again by authors to find out exact figures (Tables 1 and 2).

Discussion

Regarding the variety of case studies, it can be realized that conical and polyhedral domes are so various and have different morphological features and plan forms in each era. Generally, Islamic domes present

¹Sunni: One of the two main branches of Islam, differing from shia in its acceptance of the first three caliphs.

²Shia: One of the two main branches of Islam, regarding Ali, the fourth caliph, as Muhammad's first true successor.

³Hedrons: Denoting geometrical solids having a specified number of plane faces or denoting geometrical solids having faces of a specified shape.

a wide variety of sizes and types, but some geometric properties were repeatedly used in their composition designs. Nevertheless, no two samples are exactly the same [7].

Through geometrical analysis of case studies, the authors have noticed the value near the $\sqrt{2}$ Ratio (with an approximation of less than 0.08) or in some cases the value near the $\sqrt{3}$ Ratio (with an approximation of less than 0.04) for plan's external diameter to internal diameter (O) for most case studies. In architecture dating back to prehistory, particularly in Islamic arts and architecture, the most important geometric proportional systems are: the proportions of the golden mean and the primary three proportional roots $\sqrt{2}$, $\sqrt{3}$ and $\sqrt{5}$, on which all Islamic arts and architectural forms, especially their geometric pattern design, are based [1]. The usage of $\sqrt{2}$ and $\sqrt{3}$ Ratio and modules in Iranian architecture in pre-Islamic era and Islamic era show an exact system of proportions which have been widely used [24]. These Ratios can be seen and followed in the below diagrams of superimposed squares and circles (Figures 6 and 7).

In the above diagrams, the circle plays a major role in defining and designing plans. The circle is an obvious example of a basic geometry, constituting all the proportional geometries inherent in traditional architecture. The circle of Unity is the most significant form, for it contains a circumference revolving around a fixed center. It is therefore

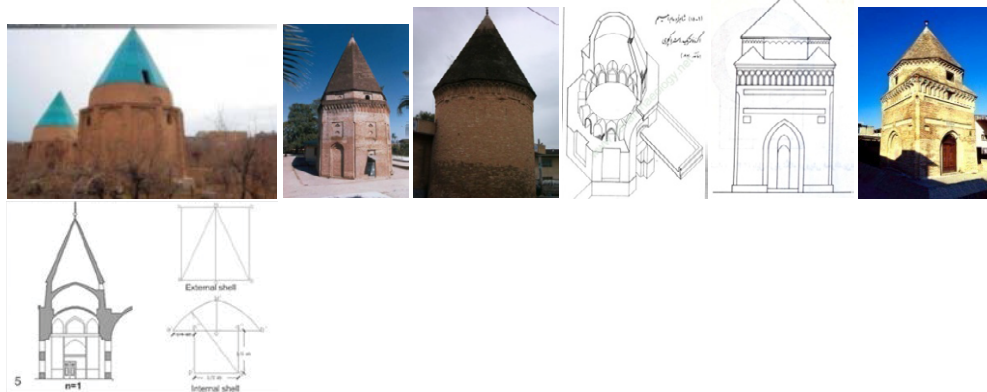


Figure 3: Case studies of Timurids era. From left to right: Khaje ali safi [photo: 2]. Abbas shrine and Qasem [photo: <http://mapcarta.com/27004910/Photos>]. Ebrahim shrine [photo: 2]. Shams tabarsi [photo: 35]. Zeinolabedin [photo: <http://mcto.blogfa.com/1391/03>]. Bayazid bastami [photos:6].

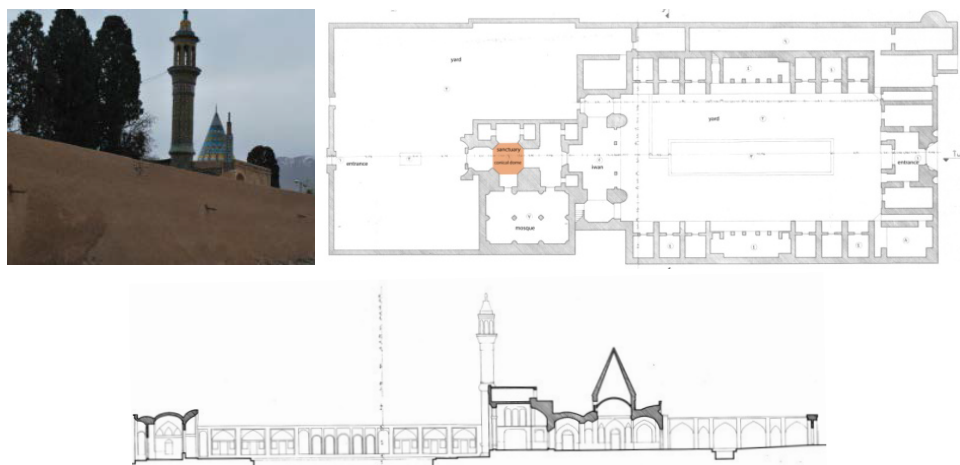


Figure 4: Shahzade Ebrahim in Kashan. From left to right: Shahzade Ebrahim among a lot of cypresses and near one of Kashan's old gates [photo: by authors]. Plan of the complex. section of the complex. Drawing: Cultural Heritage.



Figure 5: Case studies of other eras. From left to right: a and b) Mirmeshane [photo: 11], c and d) abulolo [photo: authors], e) Shahzade Ebrahim [photo: authors].

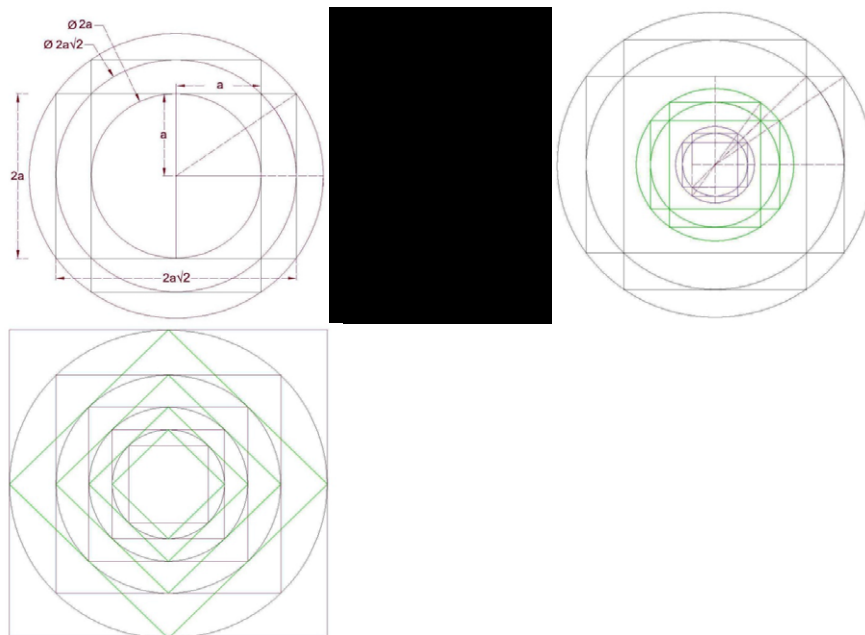


Figure 6: Diagrams of superimposed squares and circles showing $\sqrt{2}$ proportion.

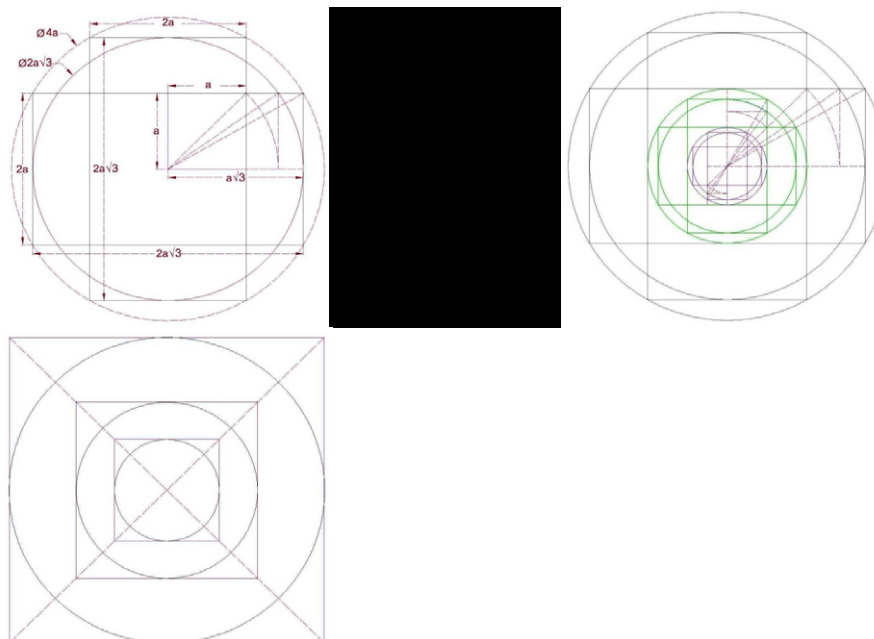


Figure 7: Diagrams of superimposed squares and circles showing $\sqrt{3}$ proportion.

Case Studies (Towers, Tombs, shrines)	Place	Era	Plan			Parts					Ref
			Exterior Plan- (number of divisions)	Interior Plan	bench	Load Bearing System	Transition Tier	Drum	Shell		
									Exterior Shell	Interior Shell	
Qabus Dome	Gorgan	1007	circle- 10	circle	no	cylinder			one shell and conical		[2]
Mehmandust	Damqan	1097	circle- 22	circle	yes	cylinder					[31]
Toqrol Tower	Rey	1139	circle- 24	circle	yes	cylinder			conical		[31]
Surkh Dome	Maraqe	1147	Square	square	yes	Cubic	octagon		octagonal pyramid	semi-spherical	[6,8,31]
Seh Gonbad	Oromie	1180	Circle	circle	yes	cylinder			conical	saucer	[6,23,31]
Kabud Gonbad	Maraqe	1196	Decagon	decagon	yes	decagonal prism		10-sided prism	pyramid		[10,31]
Damavand Tower	Damavand		Octagon	circle	yes	octagonal prism			octagonal pyramid	saucer	[31]
Above case studies are of Seljuks Era, 1038-1194 A.D											
Alla eddin Tower	Varamin	1289	Circle	circle	no	cylinder			conical	semi-spherical	[2,5,23]
Sheikh abddolsamad	Natanz	1325	Square		no	octagonal prism	octagon	Octagonal prism	octagonal pyramid	saucer	[2,5,18]
Hamdollah Mostofi	Qazvin	1340	Square		no	Cubic	octagon	cylinder	conical	semi-spherical	[5,16]
Khaje asilodin	Qom	1340	12-sided	octagon	no	12-sided prism		16-sided prism	One shell and 16-sided pyramid		[2]
Ali ebne jafar	Qom	1346	Octagon	square	no			12-sided prism	12-sided pyramid	semi-spherical	[2]
Baba qasem	Esfahan	1350-1400	Square	square	no	Cubic		Octagonal prism	One shell and octagonal pyramid		[6]
Radkan tower	Radkan		12-sided	octagon	no	cylinder			conical	semi-spherical	[28,29]
Above case studies are of Ilkhanids Era, 1256-1353 A.D											
Khaje ali safi	Qom	1398	12-sided	octagon	no	12-sided prism		16-sided prism	16-sided pyramid		[2]
Abbas shrine	Qom	1406	Octagon	octagon	no	octagonal prism	octagon		octagonal pyramid	semi-spherical	[4]
Ebrahim shrine	Qom	1412	Octagon	square	no	octagonal prism		16-sided prism	16-sided pyramid	semi-spherical	[2,3]
Shams tabarsi	Amol	1427	Square	square	no	Cubic	octagon		Square pyramid		[23,34]
Qasem Shrine	Sari	1466	Circle	circle	no	Cylinder	12-sided prism		decagonal pyramid		[4]
Zeinolabedin shrine	Sari	1484	Square	square	no	Cubic	octagon		octagonal pyramid	semi-spherical	[4,16]
Bayazid Bastami complex	Natanz	1516	Square		no	Cubic		cylinder	Triple-shell and conical	saucer	[5,6]
Shomali tomb tower	Qom		Octagon	octagon	no	octagonal prism			16-sided pyramid		[2]
Above case studies are of Timurids Era, 1370-1510 A.D											
Mirneslane Shrine	Kashan	1583	octagon	12-sided	no			16-sided prism	16-sided pyramid	semi-spherical	[11]
Abulolo Tomb	Kashan	before Safavi				decagonal prism		20-sided prism	20-sided pyramid		authors
Shahzade Ebrahim	Kashan	qajar	Part of a complex					16-sided prism	16-sided pyramid	saucer	authors

Table 1: Shows the descriptive features of case studies.

extremely important to understand that all other geometric shapes can be determined from a circle, from which come the full polygons, including series of roots and proportions [1]. Some researchers stated that the use of the circle is a way of expressing the Unity of Islam. According to this doctrine, the circle and its center is the point at which all Islamic patterns begin [6]. Circular plan in Islamic architecture represents one of the most important aspects of life which is unity and perfection. Circle is the symbol of holiness and the only perfect figure able to express God' glory [25]. Squares, pentagons, octagons, decagons and all other polygons can be drawn based on surrounded or inscribed circles. That is why diagrams have been prepared based on circles and squares.

Having examined O Ratio, we turn to M and N Ratios. These two Ratios do not show any special relationship unless M Ratio reveals the fact that height to exterior diameter Ratio has increased to some extent over time. It shows a revolutionary process in the proportions of conical domes.

Conclusions

Conical, polyhedral and pyramidal domes are among distinct features of Iranian domes which play an important role in the development of tomb towers and shrines. In Iranian architecture [26], geometry and proportions provide developments for constructions and designs of conical domes. Through geometrical analysis of case

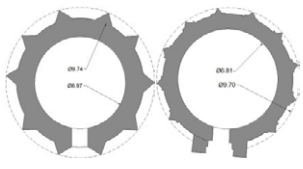
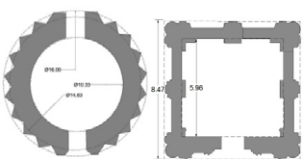
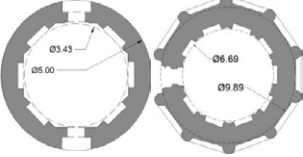
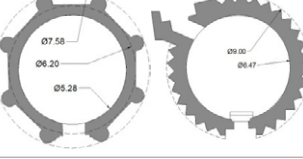
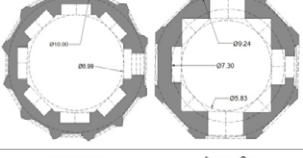
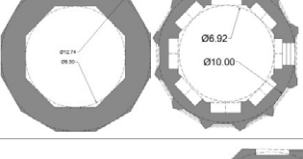

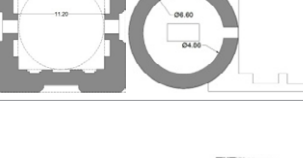
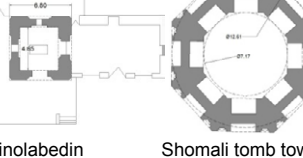
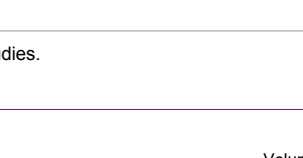

Case Studies	Dimensions and Proportions							Plans- All plans' drawings: authors	Ref
	Height of Bench	Height	Interior diameter = I	Exterior diameter=E	N=Rise/ span	M=H/E	O=E/I		
Seljuks Era, 1038-1194 A.D									
Qabus Dome		57	6.9	9.7	1.3	5.8	~1.40		[2]
Mehmandust (right plan)	1	14.5	6.8	9.7		1.49	~1.42		[31]
Toqrol Tower		21	10.3	14.8		1.418	~1.43		[31]
Surkh Gonbad (right plan)	1.5	10	5.9	8.4		1.2	~1.42		[6,8,31]
Seh Gonbad	1.5	13	3.4	5		2.6	~1.45		[3,23,31]
Kabud Dome (right plan)	2.3		6.7	9.8			~1.46		[10,31]
Damavand	2	10	5.25	7.5		1.6	~1.44		[31]
Ilkhanids Era, 1256-1353 A.D									
Alla eddin Tower (right plan)		18	6.45	9	1	2	~1.40		[2,5,23]
Khaje asilodin		20	6.9	10		2	~1.44		[2]
Ali ebne jafar shrine (right plan)		22.5	5.8	9.25	1	2.4	~1.59		[2]
Radkan Tower		25.6	8.5	12.7	0.75	2	~1.49		[29,28]
Timurids Era, 1370-1510 A.D									
Khaje ali safi (right plan)		20	6.9	10		2	~1.44		[2]
Abbas shrine		20	8.9	13.3		1.5	~1.49		[4]
Ebrahim shrine (right plan)		14.6	5	8.6	1	1.70	~1.72		[2,3]
Shams tabarsi		32	11.2	16.7	0.55	1.9	~1.49		[23,35]
Qasem Shrine (right plan)		15	4.9	6.6		2.3	~1.35		[4]
Zeinolabedin		20	4.65	6.8		2.9	~1.46		[4,16]
Bayazid Bastami complex		20		8	1	2.5			[5,6]
Shomali tomb tower		12	7.1	12.6	0.95		~1.77		[2]
Other eras									
Mirnesane		20	7.3	10.2	1	2	~1.4		[11]
Shahzade Ebrahim		14		4.8	1.2	2.9			Cultural heritage

Table 2: Shows the quantitative features of case studies.

studies, the authors have noticed the value near the $\sqrt{2}$ Ratio and $\sqrt{3}$ Ratio for plan's external diameter to internal diameter (O) for most case studies. This Ratio is the most important one since it reveals the form and dimensions of the plan. Diagrams have been presented showing these Ratios by superimposed squares and circles. Circle is the most significant form of these diagrams for its basic geometry. All samples' plans can be designed based on these diagrams.

Finally, it can be concluded that recalling mathematics and proportions to Iranian architecture [27-30] is of great importance and value for a better connection between past and present architecture. A comprehensive understanding of conical domes and their proportions can set the stage for standards which strengthen such a connection.

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