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Development of a Mathematical Model to Characterize the Oil Production in the Federal Republic of Nigeria

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Abstract

The study deals with the development of a mathematical model to characterize the oil production in Nigeria. This is calculated by initiating the dynamics of oil production in million barrels revenue plan cost of oil production in million naira and unit cost of production from 1974-1982 in the contest of the federal Republic of Nigeria. This country export oil to other countries as well as importing specialized crude. The transport network from origin/destination tij to pairs is taking into account simulation runs, optimization have been considered in this study.

Keywords: Mathematical oil model development dynamics; Characterization barrels

Introduction

Oil is the lifeblood of the modern industrial economy. It fuels the vast majority of the world's mechanized transportation equipment such as automobiles, trucks, airplanes, trains, ships, farm equipment, the military, etc. World economic growth is largely affected by rising production of oil for the last century [1,2]. Besides, this oil production rate influences the economy of the exporting countries. In an era of globalized business operations, large and small oil and gas producers alike strive to foster their profitability by improving the agility of exploration endeavours and the efficiency of crude oil production, storage and transport processes. Consequently, they all face numerous and acute challenges: ever-increasing international production, global competition, price volatility, policies dictating operational cost reductions – most of all, aggressive financial goals (revenue, cash flow and profitability) and strict environmental constraints. All the foregoing considerations should be incorporated and revised at will if the generality of production optimization algorithms is to be ensured. Their straightforward translation of these considerations to explicit mathematical objectives and constraints must then yield optimal oilfield planning, design and operation policies. The present paper provides a summary of a strategy that can facilitate interfacing and seamless integration of equation-oriented process modeling and reservoir computational fluid dynamics (CFD), in order to include the dynamic behaviour of reservoirs into oil and gas production models. Oil and gas production optimization is standard practice in the extraction and processing industry at the strategic as well as the tactical level, in order to meet fluctuating demand and maximize profit. As oil and natural gas prices continuously soar to previously unimaginable levels, producers must strive to satisfy global demand: this is vital to ensure social needs as well as international stability. Dynamic oil and gas production systems simulation and optimization is a research trend which has the clear potential to meet the foregoing challenges of the international oil and gas industry and thus assist oil and gas producers achieve their business goals while meeting energy needs. Previous work [3,4] has addressed successfully research challenges in this field, using appropriate correlations (Peaceman model) for two-phase flow of oil and gas in production wells and pipelines, and making a series of assumptions: the fundamental one is the steady-state assumption for the reservoir model, and it based on the vast timescale differences between different spatial levels (consider that, while the dynamics of oil and gas reservoirs have characteristic times in the order of weeks, the respective ones of pipeline networks are in the order of minutes, and the time horizon of the production optimization problems is in the order of days). A mathematical model is a description of a system using mathematical concepts and language. The process of developing a mathematical model is termed mathematical modelling. Mathematical models are used not only in the natural sciences (such as physics, biology, earth science, meteorology) and engineering disciplines (e.g. computer science, artificial intelligence), but also in the social sciences (such as economics, psychology, sociology and political science); physicists, engineers, statisticians, operations research analysts and economists use mathematical models most extensively. A model may help to explain a system and to study the effects of different components, and to make predictions about behaviour. Mathematical models can take many forms, including but not limited to dynamical systems, statistical models, differential equations, or game theoretic models. These and other types of models can overlap, with a given model involving a variety of abstract structures. In general, mathematical models may include logical models, as far as logic is taken as a part of mathematics. In many cases, the quality of a scientific field depends on how well the mathematical models developed on the theoretical side agree with results of repeatable experiments. Lack of agreement between theoretical mathematical models and experimental measurements often leads to important advances as better theories are developed.

Existing models have fared poorly in predicting global oil production. Even for models that are commonly thought to be successful, after-the-fact interpretation of the success or failure of a predictive effort is not easy (recall the discussion above of Hubbert's successful prediction) Simple curve-fitting models can provide a first-order understanding of future production, assuming a given level of URR and no significant shocks to the system (e.g., demand continues to

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grow at rates within historical ranges). Such models are likely sufficient to predict the decade of peak production for an estimate of URR. The mathematical logic here is that consumption is so high during the years of peak production that minor variations in URR, or minor deviations due to political or economic factors, will not serve to significantly affect the date of the peak. Unfortunately, such a conclusion is often of little practical use: major disruptions (e.g., the oil crises of the 1974 and 1979), or major errors in URR estimates have occurred in the past, and could occur again. More-detailed mechanistic models (e.g., bottom-up, econometric), exhibit greater fidelity in reproducing historical data and are therefore likely more useful for near term predictions. But this advantage likely wanes for long-term forecasts because they are no less "brittle" with respect to uncertainties than other model types.

Nature of Problem of Study

Development of mathematical model characterizing oil production in Nigeria 2004

At the department of Electrical Engineering energy system group 2004 he initiated this study and analyzed it in the following sequence as thus [1].

The dynamics of petroleum/oil production model is given by the following notation:

$\dot{Z}$=E [D-C]z (1)

where z: the oil production in million barrels.

E: Constant.

D: Revenue from oil.

C: Cost of production of oil (cost include production and transport).

$\dot{Z}$: Rate of change of oil production.

$D_j=\Sigma_i S_{ij} Pi$ (2)

where Sij is the flow (quantity) of oil between market I and production point j (i: the country to which oil is exported, j is the place of production which may be Nigeria or a specific oil field in Nigeria).

P: is the unit price at market I (the unit price of oil in the country to which oil is exported).

$C_j=Z_j (C_j+rd_{ij})$ (3)

where C_j is the unit cost of oil production,

r is the unit cost of transport of oil d_{ij} is the distance from j to I (d_{ij} is the distance from port of origin to port of destination).

C=zf, where $f=(C_j+rd_{ij})$

$\dot{Z}$=E (D-Zf) z=E D Z-E f Z^2

$\dot{Z}$-E D Z+E f Z^2=0, Non-linear equation in z

$\dot{Z}$=E [D-C] z

By differential equation we have dZ (τ)=E [D-C] z(τ)/dτ

Difference equation notation gives $\dot{Z}$(τ)-E [D-C]Z(τ)=0

By discretisation the following notation is given as:

$\dot{Z}$(dτ)=Zτ(kτ)-Z(τk)/Δτ

In Z(τk+1)-E (D-C) Z(τk) Δτ=0

Z (τk+1)-(E (D-C) Δτ+1) Z(τk)=0

Z (τk+1)-(E (D-C) Δτ+1) Z(τk)

Z (τk+1)=Z(τk)

or Z(k+1)=X Z(τk), where x=E (D-C) Δτ+1

Since Δ is constant, x is constant

Solution of this first order discrete equation

Z(k)=$(+x)^k$ Z(0), Z(0)=base year production

Actually, the equation is non-linear, and therefore the above analysis needs modification

For the given data Z(k+1)=0.965 Z (k),

For K=1, Z(1)=-x Z(0)=-0.965 × 823 × 10^6

For k=2, Z(2)=$(+x)^2$ Z_4 (0),=x^2Z (0)=0.93 × 823 × 10^6

Z(3)=+x^3Z(0), Z(4)=x Z(0), Z(5)=+Z (0)

This shows that the response diminishes over a period of time.

z(k+1)=x Z(k)

k=0, 1,2,3

∴ Z(1)=x Z(1)=x [x_2Z(0)]=x ^{2}Z(0)

Z(3)=Z(2)=x [x z(0)]

z(3)=x ^{3}z(0)

∴ z(k)=x ^{k}z(0) Solution

E(τ)=E [D-C] z(τ)

=E [D-Z(τ) (Cj+rdij)] z(τ)

=E D Z(τ)-E Cj Z^2(τ)-Erdijz^2 (τ)

Z(τ)+E[Cj+rdij] Z2(τ)-E D Z(τ)=0

Non-linear differential equation to be discretised Earlier we have defined D=$\Sigma_i S_{ij} P_i$: revenue from oil

If the entire oil which has been extracted is exported then $\Sigma_i S_{ij}$=Z, the oil produced. However, there is always a lesser amount of export than the amount extracted. So in general we have $\Sigma_i S_{ij}$</z similarly, there may be many destinations to which oil is exported, that too from many regions. so we may write more generally:

C=$Z^1Cj^1+Z^2 Cj^2+Z^n Cj^n$+r [$Z^{-1}$ di^1j+Z^{-2} $di^2j+\ldots Z^{-m}$ d^m ij]

where z=$z^1+z^2+\ldots+z^n+z^d=z^{-1}+z^{-2}+\ldots z^{-m}+z^d$

and $z^1/=z,^1$ $z^2/=z^{-2}$ …etc, and ^{zd} is amount of oil for domestic use. But $z^1+z^2+\ldots z^n=z^{-1}+z^{-2}+\ldots z^{-m}$

C^1j: Unit cost of oil production at site 1 in origin j.

C^2j: Unit cost of oil production at site 2 in origin j.

z^1: Amount of oil production at site 1.

z^2: Amount of oil production at site 2.

z^n: Amount of oil production at site n.

d^1ij: Distance between destination 1 in ½ and origin j.

d^2ij: Distance between destination 2 in ½ and origin j.

d^mij: Distance between destination m in I and origin j.

r: Unit cost of transport of oil. From the above consideration, we see that

$Z(\tau)Cj=z^1Cj^1+z^2Cj^2+\ldots Z^nCj^n$ and

$Z(\tau)\ dij=r\ [z^{-1}dij^1+z^{-2}dij+\ldots z^{-m}d^mij]$

However, let us consider the nonlinear form of the equation

$\dot{z}(\tau)+E(Cj+rdij)\ z^2(\tau)-E\ D\ z(\tau)=0$

Let us assume that $z(\tau k)=z(\tau k+1)-z(\tau k)/\Delta\tau$

Substituting this value, we get

$z(\tau k+1)-z(\tau k)+E\Delta\tau\ (Cj+rdij)\ z^2\ (\tau k)-E\ \Delta\tau\ D\ z(\tau k)\ 0$

or $z(k+1)+(1+E\Delta\tau D)\ z(k)+E\Delta\tau(Cj+rdij\ z^2\ (k)=0$

or $z\ (k+1)+az^2(k)-bz(k)=0$

where $a=E\Delta\tau(Cj+rdij)$, $b=(1+E\ \Delta\tau\ D)$

solution of $z(k+1)+a\ z^2(k)-b\ z(k)=0$ (4)

in any given situation $z\ (0)$=constant, $k>/0$

then for k=0, $z\ (1)=-a\ z^2\ (0)+b\ z(0)$

or $z\ (1)=z(0)\ [b-a\ z(0)]$ (5)

for k=1, $z(2)=-a\ z^2(1)+b\ z\ (1)$, $z\ (2)=z\ (1)\ [b-a\ z(1)]$

substituting for z(1) from the previous step we get

$z(1)=z(0)\ [b-a\ z\ (0)]\ [b-a\ \{z\ (0)\ (b-a\ z(0))\}]$

$=z(0)\ [b-a\ z\ (0)]\ [b-abz\ (0)+a^2z^2(0)$

$=z(0)\ [b-a\ z\ (0)]\ [b-abz\ (0)-ab\ z(0)+a^2b\ z(0)-a^3z^3(0)]$

$=z(0)\ [b^2-(ab^2+ab)\ z(0)+2a^2b\ z^2\ (0)-a\ ^3z^3(0)]$

$z(2)=b^2\ z(0)-(ab)\ (b+1)\ z^2\ (0)+2\ a^2b\ z^3(0)-a\ ^3z^3\ (0)$ (6)

For k=2, $z(3)=-az^2\ (2)+bz(3)$

Substituting for z(2) from the previous step we get:

$z(3)=-a\ [b^2z(0)-ab\ (b+1)\ z^2(0)+2\ a^2b\ z^3(0)-a^3z^4(0)]^2+$

$b[b^2z(0)-ab\ (b+1)\ z^2(0)+2\ a^2bz^3(0)-a^3\ z^4(0)]$ (7)

Which can be simplified. Similarly we can go for

$k=4, 5, \ldots z(4)-az^2(3)+bz(3)$

Nature of E, Δτ: In the original equation

$\dot{z}(\tau)=E(D-C)\ z(\tau)$

We see that if the difference (D-C) is small, that is not much earning (due to lower price of crude oil and higher cost of production and transport), we prefer to keep the rate of change of oil extraction (or production) at a lower value. This depressing situation can further be depended if we choose E<1. On the other hand, if the difference (D–C) is high, we would like to produce more, and in such a situation if we choose E>1, then we may tend to produce more. So value of E can vary either to depict depressing situation or "bouncing spirit". ideally E=1.

Δ is constant to indicate differential in terms of difference. Its value may be equal to 1 in a time scale whose unit is a year.

$z\ (k+1)+a\ z2\ (k)-b\ z\ (k)=0$ (8)

For k=0, $z\ (1)=b\ z(0)-a\ z^2(0)$ (9)

For k=1, $z(2)=bz(bz(1)-a\ z^2(1)$ (10)

For k=2, $z\ (3)=b\ z\ (2)-a\ z^2\ (2)$ (11)

For k=3, $z(4)=b\ z(3)-a\ z^2\ (3)$ (12)

For k=n, $z\ (n)=b\ z(n)=a\ z(n)^2$ (13)

Where $a=E\ \Delta\tau\ (Cj+rdij)$

$b=(1+E\ \Delta\tau\ D)$

Taking E=1, Δτ=1 we get

$a=(Cj+rdij)$=unit cost of production

$b=(1+D)$=average revenue earned.

$z(n)=b\ z\ (n-1)-a\ z^2\ (n-1)$

$z(n)=z\ (n-1)\ [b-a\ z\ (n-1)]$

Let $\times$ b-a z (n-1)

$x=(1-E\ \Delta\tau\ D)-E\ \Delta\tau\ (Cj+rdij)\ z\ (n-1)$

$x=[1-E\ \Delta\tau\ \{D-(Cj+rdij)\ z\ (n-1))\}$

$x=[1-E\ \Delta\tau\ (D-C)]$,.. Cj=rdij.

Data Analysis

The data for oil production in million barrels represented as ż revenue obtained from oil in million naira D cost production of oil in million naira, unit cost production in million naira from 1974-1982 is shown in Table 1. This is calculated as thus oil production Oil production for (974 to 1982) by substitution [5,6].

Years	Oil product introduces	Revenue from oil million	Cost of production of oil MN	Unit of cost product MN
	Z	D		
1974	823.318	5365	0.724	8.71×10^{-4}
Z(0)				
1975	651.507	4 555	0.706	1.083
1976	758.055.380	6.321	6.733	9.66914
1977	766.053.944	7.072	0.657	8.57046
1978	692.269.121	5461	0.834	1.20475
1979	842.474	10.166	0.736	8.736.7
1980	754.620,497	13.523	0.931	1.2337
1981	381.395	610.453	1.315	3.44767
1982	474.500 000	9207	1.359	2.86406
Total	6.144.192341	25.2355	13995	45.5195
Average	682.688	2.80395	1.555	5.05772

Table 1: Oil production revenue for oil and cost of production.

Results

∞=[I-E Dt (D-C)]

Taking tij=rdij

Z (I)=b z (o)-a z^2 (o)=Z (o) {b-a (o)}

Z (o)=823.317 $\times 10^6$ barrels produced in the base year

B=8013.67 $\times 10^6$ Revenue earned

a=(14.901 $\times 10^{-4}$) MN=N14.901 $\times 10^{-}$N

1.294NM

Z (I)=8013.67 $\times 10^6 \times$ 823.317 $\times 10^6$ (823.317 $\times 10^{12}$)

1.294.

=659 77 90.7 $\times 10^{12}$ (677850.88 $\times 10^{-12}$

Z ($1^{>}$)=6597790.7

Z (I)=65967806344. 4 $\times 10^8$

Z (II)=659.678 $\times 10^{16}$

$\sqrt{}$=(659 678 $\times 10^{10}$) $\times 10^6$ barrels

$\sqrt{}$Z (1)=(659. 678 $\times 10^{10}$) m barrels

Z (1)=659 678 oooooo m barrels

$\dot{z}=\sum$(5.365-0.724 $\times 10^4$) 823,317,838 for 1974

=4.35 749 $\times 10^9$

$\dot{z}=\sum$(4553-0.706) 651, 506761 for 1975

2.95756 $\times 10^9 \times$ 100=2.96 756 $\times 10^{11}$

4.35749 $\times 10^9$=6810.25095

This is calculated from the table above 1974-1982 calculated accordingly to obtain the cost of production of oil the unit cost of production are computed from 1974-1982 the total and average could be obtained as well.

Discussion and Contributions

From the findings of the study conducted by Prof Paul CN in 2004 in the oil model dynamics it is noted the oil production in million barrels from 1974-1982 remained fluctuated as well as oil revenue, cost of production and unit cost of production those not remain almost at 0 point level for cost of production. This indicated that the trend increases and falls. There are certain years, 1974, 1979 the oil production was at an increase and it became stable in 1976, 1977, 1980 almost at equilibrium. This is due to the international market mechanism of OPEC countries to pushing the quota by increasing the production rate as well as reducing the quota and prices of members states. There are a few studies which involved multipurpose inter-regional equilibrium model to consider the technological and locational choices in the petroleum industry. Such a model assumes cost minimization as an objective function. Certain exogenously given locational demands for petroleum products are needed to augment a great number of constraints with particular reference to the availability of resources. Optimal levels of foreign trade in crude oil, petroleum products are determined from the model. Furthermore, this model includes the optimal levels of various refining processes for example catalytic cracking, hydrocracking at various locations. The optimum patterns of transportation of various crude oils or petroleum intermediates and final petroleum products are also decided. The solution to the model provides optimum values consistent with the constraints on the petroleum product requirements at various locations. In addition to this the solution process furnishes an indication of the shadow prices of difference petroleum products for different locations. In general the model gives an easy to handle tool for analysing the policy implications in accordance with the cost and other coefficients applied or adopted in the model.

Conclusion

The development of mathematical model to characterize the oil production in Nigeria is successfully made. However despite the fact that Nigeria is an oil exporting country realizing huge increase in oil revenue investment in infrastructural development is not heavily reflected in the well-being of Nigerian society as a whole. The government is being informed to concentrate on the betterment of the population and use the oil money revenue in a such a way that Nigeria will benefit most. The study noted a dynamic nature of increase in oil revenue at the same time fluctuation in the oil production as well as the prize of oil. The study noted that oil revenue will help in budget management and planning for the federal government of Nigeria. Most of the time the budget allocation of the states of Nigeria is dependent on oil production. When there is decrease in oil price it affect the budget when there is increase in oil price it enhances the budget. However Nigeria exports oil to other countries and imports special crude and this dependent on international market demand on oil.

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